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Instability and convection in fluid layers: A report on Euromech 106

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The 106th Euromech Colloquium on instability and convection driven by body forces in fluid layers was held in Grenoble from 11 to 14 September 1978 with the first two authors acting as chairmen. There were sixty-five participants coming from fifteen different countries and having widely different backgrounds. Fifty-seven papers were presented during the four full days of the meeting and are discussed in this report with the purpose of giving an up-to-date view of current research in convection.

1. Introduction

Instability and convection induced by body forces have widespread interest in many branches of physics and engineering, ranging from heat and mass transfer in industrial or natural flow systems to planetary and stellar convection. Besides, the transition between different macroscopic states of a system and the fascinating idea of the emergence of order from a non-ordered state and the possible return to disorder called 'turbulence' has recently attracted the attention of mathematicians as well as theoretical and experimental physicists and this has stimulated new experiments on the Rayleigh–Bénard problem and on Taylor vortex flow. The subject of convection has usually been discussed as part of conferences on general topics such as geophysics, astrophysics, heat transfer or electrohydrodynamics but there have been rather few conferences devoted to it entirely.† In recent years significant progress has been made on many aspects of convection and it was felt that the ground was ready for profitable discussions. This situation and the activity in Grenoble on electroconvection in dielectric liquids, which has many analogies with the Bénard problem but bears in addition new interesting features, gave the original incentive for organizing the Colloquium.

The primary objective was to have an interdisciplinary meeting bringing together workers from all the different fields where body-force driven convection is of

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† Two colloquia of more limited scope may be mentioned in this context: one organized by I. Prigogine in 1973 (Prigogine & Rice 1975) and the other by J. C. Legros and J. K. Platten in 1977 (Legros & Platten 1978).

importance. Besides phenomena particular to the different fields, common mechanisms were to be stressed. The mechanisms discussed were in particular the types of instabilities and the structure of the resulting motions, their dependence on the pertinent parameters and external constraints, the further branching to turbulence, the influence of shear and convective transport. The most extensively studied problem is Rayleigh–Bénard convection considered in a large sense. This problem is relevant to a great number of applications and it is also a relatively simple system for studying theoretically and experimentally the above mentioned mechanisms. It was thus natural to devote a large part of the Colloquium to this problem. A particular effort was, however, made to include new and less known situations like: convection in liquid crystals used for imaging processes, in dielectric liquids where it has a drastic effect on the electric insulating properties, in biological systems where it is induced by living organisms and in crystal growth melts. Mention should also be made of the important problem of convection in porous media.

The programme was arranged partly with the different applications in mind and partly to allow a coherent discussion of the important mechanisms. The contributions were grouped in six sessions having the following themes:

(i) Rayleigh–Bénard convection.
(ii) Penetrative convection and effects of stable stratification.
(iii) Doubly diffusive convection.
(iv) Rotating systems.
(v) Other driving forces.
(vi) General theories and transition to turbulence.

The three major themes (considering that penetrative convection is an extension of Rayleigh–Bénard convection) (i), (iv) and (vi) were introduced by review lectures, respectively given by R. Krishnamurti (Florida State University), F. H. Busse (UCLA) and D. D. Joseph (University of Minnesota). Introductory lectures were given on subjects which emerged mainly during the last ten years: on doubly diffusive convection by P. F. Linden (University of Cambridge); on convection in liquid crystals by E. Guyon (University of Paris, Orsay); and on electroconvection by E. J. Hopfinger (University of Grenoble).

In this report the grouping is basically the same as was adopted for the colloquium. Some minor rearrangements are made in order to facilitate the discussion of related papers. The report is intended to give an up-to-date view of current research in convection.

2. Rayleigh–Bénard convection

Convection in a horizontal layer of fluid heated from below has received considerable attention during the past 15 to 20 years and it constitutes one of the most studied hydrodynamic systems. It was thus natural to open the colloquium with a session on this fundamental problem.

In a survey lecture R. Krishnamurti*† recalled first the main features of Rayleigh–Bénard convection describing the characteristics of the different flow régimes obtained at various Rayleigh (further denoted by R) and Prandtl (P) numbers (Krishnamurti

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† The papers marked with an asterisk were presented at the Colloquium.

1973). Then she turned to the stability problem in cases where there exists some asymmetry versus the horizontal mid-plan. The formation of hexagonal cells and the existence of subcritical finite amplitude convection (see for instance the review by Palm 1975). The nonlinear stability analysis based on a two-parameter expansion of the dependent variables and of the Rayleigh number (Busse 1967; Krishnamurti 1968*a*) provides good predictions for the stability domains of rolls and hexagonal cells (Krishnamurti 1968*b*). The first example of this type discussed by Krishnamurti concerned the formation of cellular cloud patterns that she ascribed to the existence in the atmosphere of large scale ascending or descending flow of low but finite mean amplitude (Krishnamurti 1975*a*, *b*). Bio-convection provides another example to which the two-parameter mathematical treatment may apply. Micro-organisms often display a negative geotactic behaviour, i.e. a tendency to swim against gravity. This induces a nonlinear density distribution and defines a small parameter. The model accounts for the instability observed in laboratory cultures of micro-organisms (see also Childress, Levandowsky & Spiegel 1975).

The analogy between instability in the linear Rayleigh-Bénard problem and second-order transition in physics has often been outlined (see for example Graham 1975). It is well known that just above the onset of convection the amplitude of the velocity varies like $(R-R_c)^{\frac{1}{2}}$ and the growth rate varies linearly with R (see Donnelly, Schwartz & Roberts 1965 in the case of the Taylor problem). M. Dubois & J. Wesfreid* (C.E.N. Saclay, Gif-sur-Yvette) reported on accurate verification of these laws and also on new results concerning the spatial distribution of motion near a lateral wall in supercritical conditions and in a slightly subcritical layer in which a finite motion is locally maintained. A generalization of the Landau expansion allowing for the spatial variations of the amplitude of the rolls gives predictions which are very well verified by experiments (Wesfreid *et al.* 1978). In both cases the influence length varies as $|R-R_c|^{-\frac{1}{2}}$ and obeys laws entirely similar to those characterizing the correlation length in critical phenomena with classical Landau behaviour. These results are consistent with those obtained from a numerical solution of the two-dimensional problem discussed by J. K. Platten (State University, Mons) and J. C. Legros* (Free University of Brussels). Their approach bears more flexibility in varying the physical conditions but because of computational limitations gave less accurate results than experiments do. Let us mention here the universality of this critical behaviour by referring to similar characteristics in the case of electrohydrodynamic instabilities in nematic liquid crystals (see Ribotta's paper, chapter 6).

The abrupt character of the onset of instability as described by bifurcation theory raises questions to which no definitive answer has been given. Some experiments (Ahlers 1975) suggest that the transition is smooth, a concept also examined theoretically by Benjamin (1978). I. C. Walton* (Imperial College, London) considered the case of imperfect sidewalls for which he prescribed the distribution of perturbed horizontal temperature derivative. The low amplitude motion forced by the sidewalls at values of $R < R_c$ is resonated when the critical Rayleigh number is approached and this smoothes the transition (Hall & Walton 1976). The emphasis on the smooth transition in contrast to the sharp bifurcation of the traditional stability analysis is mainly of mathematical interest. Although end effects are sometimes important, especially in the Taylor vortex case, they have not yet been modelled in a realistic way and their influence appears to be decreasing beyond the point of bifurcation.

Cumulus convection is an important heat transport process in the atmosphere. In attempting to model interactions between large-scale and small-scale motions J. Steppeler* (German Weather Office, Offenbach) used the Ritz–Galerkin method with a small number of judiciously chosen test functions. This technique gave satisfactory results when applied to rolls in Rayleigh–Bénard convection (Steppeler 1978) and was extended to the case of cumulus convection in a simple model where the diameter of the cloud depends on the height only.

Astrophysical convection is intrinsically three-dimensional and some features which would not be exhibited by rolls can however be found on a model of a cylindrical convection cell. C. A. Jones (University of Newcastle-upon-Tyne) and D. R. Moore* (Imperial College, London) focused on the stability of the axisymmetric flow determined previously (Jones, Moore, & Weiss 1976). At high enough Prandtl numbers ($P \gtrsim 0{\cdot}1$) the instability modes for these cells do not substantially differ from that characterizing the two-dimensional rolls. For $P \lesssim 0{\cdot}1$ a new mode appears which bears some relation to the shear instability of a vortex ring. It was suggested that this mechanism of instability may have a relevance to the exploding granule phenomenon observed in the solar atmosphere.

The evolution of the flow in a rectangular box as the Rayleigh number is raised is usually studied by visualization techniques or/and velocimetry. H. Oertel* (University of Karlsruhe) has developed a differential interferometric method which provides detailed information on the temperature field (Oertel & Bühler 1978). He reported on the different convection régimes observed in time when a constant heating rate is applied at the bottom plate. In nitrogen ($P = 0{\cdot}7$) the time-dependent flow changes in nature when the heat flux exceeds some threshold value and passes from oscillatory to wave-mode régime most likely triggered by the lateral walls. The influence of superimposed rotation was also examined. The experiments did not reveal any discrepancy between the observed onset of instability and the predictions of theory (Chandrasekhar 1961) even at high Taylor numbers. Oertel then showed a film which demonstrated the various time-dependent régimes and the change in flow pattern as the Taylor number is increased (see figure 1, plate 1).

In spite of numerous studies on time-dependent convection there is not yet a clear picture of the sequence of events occurring when R is raised. For high Prandtl number convection in particular there are noticeable disagreements between different researchers (Krishnamurti 1970; Willis & Deardorff 1970; Busse & Whitehead 1974). J. A. Whitehead* (Woods Hole Oceanographic Institution, Massachusetts) reported on recent results showing that, in a large domain of Rayleigh number, carefully initiated disturbances induce bimodal stable convection whereas random initial conditions lead to oscillating spokeshaped convection. Thus, transition to time dependent convection depends on the realized convection and not only on R and P. This transition occurs gradually in that the fraction of the convective layer exhibiting time dependent flow increases with R over an extended range of R (Whitehead & Parsons 1978). A Fourier analysis of the temperature measured at a given point showed two discrete frequencies corresponding to oscillations on a background induced by the gradual shifting of the cells. The mechanism of the oscillations preceding and following the emergence of thermals from the boundary layer is not completely clear.

The paper presented by D. J. Tritton* (University of Newcastle-upon-Tyne) concentrated on the problem of the emergence of thermals from the thermal boundary layer, as the main feature of the motion obtained when heating from below a very thick layer of a very high Prandtl number liquid. The phenomena observed can be interpreted in terms of the boundary-layer model of Howard (1966).

The following communication by Z. Peradzynski* (Institute of Fundamental Technological Research, Warsaw) was related to the oscillations in the Rayleigh–Bénard problem. It concerned the flow generated in a large gas container by a small plasma volume maintained by a laser. The resulting continuous vertical plume exhibits an oscillatory instability of a well-defined frequency. A simplified analysis accounted very well for the observed oscillation (Mucha, Peradzynski & Baranowski 1977).

The combined effect of unstable thermal stratification and shear flow has received much attention during the last decade (Kelly 1978). Above the critical Rayleigh number the secondary flow organizes in the form of longitudinal rolls with axes in the direction of the mean flow. R. M. Clever* (UCLA) presented the results of the linear stability analysis of these convection rolls for an inclined layer and a Couette flow. In both cases, for low enough shear, the instability mechanism is not altered by the longitudinal flow. As the shear is increased, a new mechanism arises, which is not encountered in horizontal layers without flow. It is the stationary wavy instability quite similar to that observed for Taylor vortices (Clever & Busse 1977; Clever *et al.* 1977). Reasonably good agreement was found with the experimental results of Hart (1971).

The experimental study by W. M. Schinkel* (University of Technology, Delft) was concerned with convection in inclined layers of finite aspect ratio (of order 10). He was mainly interested in the heat flux as a function of the inclination angle θ. Using a holographic interferometer, W. M. Schinkel was able to determine the local heat transfer and to study the different flow régimes. Two domains of low and high inclination can be distinguished, the interferograms looking like those for respectively horizontal ($\theta \lesssim 30°$) and vertical ($50° \lesssim \theta \lesssim 90°$) layers. The overall heat transfer takes a minimum value at an intermediate inclination (40–60°) the value of which depends on the aspect ratio.

The last two papers of this session were devoted to problems in porous media. J. P. Caltagirone and A. Mojtabi* (Laboratoire d'Aérothermique, Meudon) considered the phenomena occurring in the region between two concentric horizontal cylinders and examined the stability of the two-dimensional basic flow. Above a critical Rayleigh number which depends on the geometrical parameters, instability sets in, leading to three-dimensional convection and an increase in heat transfer (Caltagirone 1976). The critical conditions were theoretically determined by both linear theory and energy method.

The question of the possible thermal convection in snow layers was the topic of the communication by E. Palm* (University of Oslo). The existence of a convective motion could explain why the flux of water vapour is sometimes ten times the diffusive flux. This enhanced transport accelerates the metamorphosis of the snow and this may facilitate slab avalanches. When the air temperature has been very low for a long time convection can occur in thick layers (of the order of 1 m) of high enough permeability as frequently encountered in old snow. The occurrence of thermal convection has not only a bearing on the transport of water vapour, but also on the insulating power of the snow.

3. Penetrative convection and effects of stable stratification

The motivations for studying penetrative convection vary from ice formation or melting to convection in the lower atmosphere and in stars. In these situations a stable region is adjacent to an unstable layer and the convective motions tend to penetrate into the stable regions.

The convective instabilities in a cylindrical column of water of diameter D and variable height h, between 0 and 4 °C as well as 0 and 8 °C, were studied experimentally by M. A. Azouni* (Laboratoire d'Aérothermique, Meudon). She reported that in the case of 0–4 °C the temperature taken at a fixed point along the cylindrical wall shows regular oscillations when (Azouni 1977):

$$0{\cdot}86 < h/D < 1{\cdot}14, \quad 8{\cdot}53 < R/R_c < 12{\cdot}4.$$

Higher harmonics appear for higher aspect ratios and/or Rayleigh numbers. In the penetrative situation (0 °C below and 8 °C above) the oscillations are similar in the convecting layer but appear damped (out) in the stable region. From temperature measurements she inferred that in the steady state the mode is antisymmetric. It is of interest to mention here the related problem of convection in water over ice in plane layers which has been studied fairly extensively (see, for instance, Adrian 1975).

The modal equation technique developed by Gough, Spiegel and Toomre (1975) has been used by J. Latour, J. Toomre and J. P. Zahn* (Nice Observatory) to study the penetration depth from a convective plane into an adjacent stable region. Two-dimensional and three-dimensional convection modes have been studied and Zahn reported that the strongest penetration occurs for three-dimensional motions. The maximum penetration depth was found to be comparable to the thickness of the convective layer. In applying the analysis to stars astrophysicists are faced with the difficulty that no well-defined boundaries exist and that the Nusselt number cannot be defined easily. Zahn also reported that the heat flux depends only weakly on the convection mode, i.e. there is little difference between two-dimensional rolls and hexagonal cells unless the Prandtl is much less than unity. The penetration depth on the other hand depends strongly on the mode. In applying these models the variation of cell-size with Rayleigh number must be taken into account.

In stars the Rayleigh number varies from 10^{12} to 10^{20}, the Prandtl number is of order 10^{-6} to 10^{-9} and generally non-Boussinesq effects are of importance. Using again the modal technique but an anelastic model (Gough 1969; Latour *et al.* 1976), J. M. Massaguer (University of Barcelona) and J. P. Zahn* studied the anelastic convection in a plane layer from the onset of instability up to Rayleigh numbers of about $10^3 R_c$. Hexagonal planforms were used, retaining only one single mode. As was pointed out during the conference, if only one single mode is used in penetrative convection, there is a risk of resonance (Moore & Weiss 1973). The model showed the appearance of an inversion in the upper part of the domain due to the large pressure variations arising there. Classical mixing length theories cannot account for this. Massaguer and Zahn so far studied the stationary problem only and there is need for the study of the time dependent situation.

The following four papers were concerned with penetrative convection in the atmospheric boundary layer. First, a linear stability analysis for a semi-infinite domain was presented by P. A. Bois* (Pierre and Marie Curie University, Paris) using an asymptotic expansion with respect to a small parameter which was the ratio of a characteristic scale of the vertical motions to the scale z of the domain considered. In particular he considered an unstable region up to height z_0 and a stable region above. Numerical calculations showed cellular convection for $z \leqslant z_0$ and an exponential decay of the motion with z when $z > z_0$. The results obtained give little new information; its interest lies in the techniques used. The author claimed that it has direct applicability to atmospheric convection up to a height of several kilometres. This statement was criticized mainly because in the atmosphere mean shear effects are generally important and linear stability theory seems to be inadequate.

Measurements in the atmospheric convection layer by means of an acoustic Doppler sounder were reported by A. Weill* (Klapisz, Strauss, Van Grunderbeeck and Weill, C.R.N.S. Issy-les-Moulineaux). From the vertical doppler shift the vertical velocity variance was obtained and the acoustic backscattered intensity gave access to the inversion height. This is possible because a good correlation has been observed between the minimum in the backscattered intensity (reflectivity) and the base of the inversion height. The results obtained are in good agreement with local similarity concepts and in particular verify the relation between the local turbulent heat flux and the third power of the standard deviation of the vertical velocity fluctuations. A good estimate of the surface heat flux is obtained when the vertical velocity variance profile is extrapolated down to the ground.

Laboratory experiments have often proved to be very useful for understanding penetrative convection processes, a good example of which are the experiments by Willis & Deardorff (1974). Many features of this problem still remain unresolved and one aspect is penetrative convection above heat islands, such as a city during the early morning hours. K. M. Faust and E. Plate* (University of Karlsruhe) reported on a laboratory investigation of penetrative convection in which the heat flux was applied only to a finite area of the bottom of a stably stratified temperature gradient in water. They refer to this case as 'weak plume' in contrast to the strong plume where the heat flux is concentrated at a point source. In the weak plume the characteristic penetration height is of the order of the characteristic scale of the plume cap. The length and time scales used for correlating their data were a combination of the strong plume model and homogeneous penetrative convection. A comparison with field data showed that the weak plume model is applicable only when the heated area remains sufficiently small.

A computational model of turbulent penetrative convection was presented by J. C. André, G. De Moor, P. Lacarrère and R. Du Vachat* (Météorologie-EERM/GMD, Boulogne). It is based on the one-point space-averaged turbulent transport equations up to third order, i.e. the equations for the rate of change of third-order moments are included and closure assumptions for pressure correlations, dissipative terms and fourth-order correlations are used. The novelty of the authors' approach is apparent in the treatment of the fourth-order moments. A quasi-normal approximation together with a damping of triple correlations, called 'clipping approximation' (André *et al.* 1976) is used. The parametrization of the pressure and dissipative terms is in general similar to that of Zeman & Lumley (1976). Quite a large number of free constants must be determined in these models and this was done with reference to experimental data, previous models and computer optimization. André *et al.* applied their computational model to the penetrative convection experiments of Willis & Deardorff (1974), where the Rayleigh number was about 10^{11} and good agreement between the model and experiment was found. Let us note that this problem is dominated by convective fluxes of turbulent energy and a lower-order closure model can hardly account for these fluxes. This model was developed for application to the atmospheric inversion layer, but it could also be useful in astrophysics. The modal equations used in that context are more elegant and give more physical insight, but a turbulence model may give more realistic results.

How stable stratification can be the cause of an instability was illustrated by D. J. Acheson* (University of Oxford). Stable stratification can act as a catalyst for instability, while the energy is supplied by other sources and Acheson gave two examples. One illustrated the possible extraction of energy from a mean shear flow by an over-reflexion mechanism of internal waves in a continuously and stably stratified bounded medium, the lower part of which is at rest and the upper part is moving with horizontal velocity (Acheson 1976). The other example was of the doubly-diffusive type with unstable temperature and stable salinity gradients and it was given to illustrate the anomalous behaviour found in diffusive systems which have recently been studied theoretically (Acheson 1978).

H. True* (Technical University of Denmark, Lyngby) reported on the application of bifurcation theory to the convective boundary layer on a vertical wall with a stable temperature gradient in the ambient fluid. The stability criterion is given in terms of a critical Reynolds number which depends on the Prandtl number (Gill & Davey 1969). H. True pointed out that there exists a critical Prandtl number $P_{\text{crit}} = 12{\cdot}8$ below which bifurcation of an asymptotically stable time periodic motion takes place at the lowest critical Reynolds number. For $P > P_{\text{crit}}$ on the other hand the bifurcated solution is itself unstable and it exists for subcritical Reynolds numbers.

The final presentation of the second session was by W. Schneider and H. Keck* (Technical University, Vienna) on thermoconvective waves. This is a subject on which little is known, mainly because there are as yet few known situations where such waves have been observed. They have been demonstrated in ferromagnetic fluids and lately also in an air layer (Berkovsky *et al.* 1978). Schneider and Keck showed, by using a singular perturbation technique, that such waves can exist in any fluid layer confined between two horizontal walls with $R < R_c$. Due to buoyancy the damping of thermoconvection waves can be very weak as compared with classical thermal waves. Thermoconvective waves can also occur in a compressible medium in which case a coupling with sound waves is possible.

4. Doubly diffusive convection

There has been a considerable interest in convection phenomena in fluids where components of different molecular diffusivities make opposing contributions to the density gradient. In two-component systems there are four non-dimensional parameters which are in addition to R and P (if one is temperature) a concentration Rayleigh number and the diffusivity ratio. In doubly diffusive systems instability may set in as an oscillatory mode, or in a direct growing mode. P. F. Linden introduced the topic by giving examples of the occurrence of doubly diffusive convection in natural systems; for a comprehensive discussion of doubly diffusive phenomena see Turner (1974). A characteristic feature of doubly diffusive convection is the presence of well-mixed layers separated by thin interfaces. The way in which these layers are formed from an initially smooth salinity gradient is not well understood and H. E. Huppert and P. F. Linden* (University of Cambridge) reported on this problem. A stable salinity gradient was heated from below at a constant rate and the formation of layers was studied. The instability occurs first in an overstable mode as predicted by linear theory but as heating continues a convective régime develops and with it a well-mixed layer which grows in time (proportional to $t^{\frac{1}{2}}$) until the thermal boundary layer ahead of the convecting layer itself becomes unstable. By this process successive layers appear on top of the original layers. A movie was shown displaying this layer formation above and their destruction below. A few layers at a time could be observed that were not necessarily of equal size. A numerical model which incorporated Turner's (1965) heat and salinity flux data across the interfaces exhibited all the qualitative features observed in the experiments.

The work of T. A. McClimans and J. E. Steen* (River and Harbour Laboratory, Trondheim) was concerned with doubly diffusive fluxes across the interface between fresh water above and salt water below, both of which are at their respective freezing points. Because the freezing temperature T_F of salt water is lower than that of fresh water and because the temperature corresponding to maximum density is higher than T_F for salinity $< 24{\cdot}7\,‰$, the temperature interface is statically unstable. This can result in doubly diffusive convection and hence an increased heat loss from the fresh water leading to an increased ice growth rate. This was demonstrated in laboratory experiments. McClimans suggested that this process may occur in some Norwegian fjords.

The following three papers were in particular concerned with the Soret effect in binary mixtures. Soret convection differs from ordinary doubly diffusive convection only in the coupling between the temperature gradient and the concentration gradient which introduces in addition the ratio of thermal to mass diffusion coefficients D'/D, called Soret coefficient. J. C. Legros (Free University of Brussels) and J. K. Platten* discussed results obtained from numerical solutions of the nonlinear equations governing two-dimensional Rayleigh–Bénard convection with Soret effect in water/alcohol mixtures in a rectangular box of aspect ratio 10. The problem was similar to that treated by Platten & Chavepeyer (1977) except here rigid boundaries were considered. Computational results for negative D'/D (concentration gradient stabilizing) at $R \simeq 1{\cdot}4R_c$ showed an oscillatory behaviour with a characteristic wavelength identical to that when $D'/D = 0$. When $D'/D > 0$ and $R = 1{\cdot}6R_c$, the horizontal wavenumber decreased with increasing D'/D and Legros and Platten conjectured that when D'/D is sufficiently high one single cell might exist and this could explain why the onset of instability could not be detected by heat flux measurements (Legros, Van Hook & Thomaes 1968).

Rotation has a stabilizing effect on Rayleigh–Bénard instability and one would expect that this will also be the case for Soret instability. This aspect was studied by Antoranz & Velarde (1978), who did a linear stability analysis of a two-component Rayleigh–Bénard problem with Soret effect. M. Velarde* (Universidad Autonoma de Madrid) reported that in all situations rotation was found to increase the critical Rayleigh number. Then Velarde mentioned the Dufour effect (i.e. heat flux caused by a concentration gradient), which theoretically can be stabilizing or destabilizing, depending on the sign of D'/D and on the mode of instability. In practice, however, this effect is likely to remain always negligible. Finally, surface-tension effects were briefly discussed and this mainly for reduced gravity as it would occur in crystal growth melts in a spacecraft (Castillo & Velarde 1978).

Experimental results on Soret-driven instability in a thin slab of a very dilute solution of macromolecules in water heated from *above* were reported by M. Giglio and A. Vendramini* (C.I.S.E., Segrate, Milan). Instability (steady transition) can in this case occur only when D'/D is negative. The time evolution of the concentration gradient in the midplane was measured by making use of the deflection of a laser beam and this for slightly sub- and supercritical temperature differences ΔT. What appeared as remarkable was the very long time (a few hours) needed for the system to reach an equilibrium state (Giglio & Vendramini 1977). The critical value of ΔT corresponding to a critical concentration Rayleigh number $\tilde{R}_c$ was found to be in good agreement with linear stability theory (Velarde and Schechter, 1972). The mass flux and the perturbation growth rate showed a dependence on $\tilde{R}-\tilde{R}_c$ similar to ordinary Rayleigh–Bénard convection.

In homeotropic nematic liquid crystal layers instability can occur by heating either from below (Lekkerkerker, 1977; Guyon, Pieranski & Salan 1978) or from above (Pieranski, Dubois-Violette & Guyon 1973) and the modes observed are respectively over-stable and direct growing and this suggests a close analogy with doubly diffusive phenomena. E. Guyon introduced this subject vividly by describing the essential properties of nematics. These liquids are characterized by long-range orientational order of their elongated molecules ($\sim$ 25 Å long), which permits us to define a preferential molecular axis called 'director' and which is the cause for anisotropy in the physical properties,† as, for instance, thermal conductivity. Any deformation of the director relaxes with a definite time ($\sim 10^3$ s for a layer thickness of 1 mm) and the orientation of the molecules imposed by the boundaries plays a stabilizing role. It is in fact this relaxation time which is the analogue of the concentration diffusion time in doubly diffusive systems and these are much greater than the thermal diffusion time. A recent study by E. Guyon, P. Pieranski and J. Salan* (University of South Paris, Orsay), reported by Guyon, showed that in homeotropic nematics (director normal to the horizontal plates) heated from below oscillatory instability was observed as was predicted by Lekkerkerker. They furthermore found an hysteresis loop giving a nonlinear criterion $\Delta T_0 < \Delta T_c$ and indicating an inverse bifurcation. Experiments were also carried out with a stabilizing magnetic field which can strongly decrease the relaxation time of the director. When this time became of the order of the thermal diffusion time the oscillating behaviour disappeared. This control of the orientational relaxation time provides novel means for studying instability mechanisms encountered in double diffusive systems. Convection in liquid crystals due to other forces is discussed in chapter 6.

B. Baranowski* (Polish Academy of Science, Warsaw) considered convection in a horizontal layer of an electrolyte solution between two electrodes which is conceptually related to Soret convection in a layer heated from above. The temperature gradient which sets up the positive concentration gradient by the Soret diffusion is here replaced by an electric potential which has otherwise no effect on the instability. For example, when an electric potential is applied on a water–$CuSO_4$ solution, SO_4^{2-}

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† In particular the anisotropy of refractive index opens a wide range of display applications, the visualization resulting from externally induced orientation change in the nematic film (watch display, experimental TV screen, etc.).

ions migrate toward the anode, thus increasing the density in its vicinity. With the cathode below and the anode above the layer can become unstable. Experiments were carried out at constant current and constant electrode potential difference and the onset of convection was manifested by a change in slope of respectively the potential difference and the electric current as a function of time. The concentration Rayleigh number at which instability was 'manifest' was between 1100 and 1500 and a polygonal convection pattern was imaged on the cathode by the Cu deposit. Baranowski also reported that the approach of the steady state was by way of damped oscillations.

Marangoni instability was included in the Colloquium because it plays an important role in some convective systems and furthermore it has some features in common with some doubly diffusive processes. A broad view of Marangoni instabilities was given by H. Linde (Linde and Schwartz*, Central Institute of Physical Chemistry, Berlin). He began his talk by showing schematically the possible régimes of Marangoni convection driven by surface tension gradients due to both temperature and concentration gradients. In particular, Linde showed that wave-like oscillations or roll cells can be observed depending on whether the interface is liquid–gas or liquid–liquid. A numerical model was developed which predicted adequately the phenomena observed. The different convection regimes and the appearance of dissipative structures at high Marangoni numbers were illustrated by a movie.

D. Schwabe, A. Scharman and F. Preisser* (The Physics Institute of the University, Giessen) demonstrated experimentally that in floating zone (crystal growth) melting, Marangoni convection can be of primary importance even when gravity is not reduced. The experiments were carried out in $NaNO_3$ melts but there are reasons to believe that the results are also representative of silicon (Si) floating zone melting (Schwabe *et al.* 1977). For high Marangoni numbers convection becomes oscillatory with characteristic frequencies similar to those of temperature oscillations observed previously by other experimentalists. Usually, such temperature oscillations lead to impurities and inhomogeneities in the crystal and a knowledge of the Marangoni number at which these oscillations set in is of great importance. It was found that oscillations occur above a temperature Marangoni number of about 10^4. It should be pointed out that in the experiments carried out the melt was gravitationally stable (heated from above) and the importance of Marangoni convection as compared with Bénard convection in unstable situations remains open to discussion. Besides in some situations concentration effects due to impurities might not be negligible.

Convection in melts on a larger scale is also relevant to solar-energy storage, to metal-alloy solidification and possibly to convection in the earth mantle. J. Pantaloni described experiments carried out by Pantaloni and Bailleux* (University of Provence, St Jérôme, Marseille) in a horizontal layer of molten salts heated from below. In one case the upper boundary was a free surface whose temperature was close to the solidification point and as solidification commenced a polygonal solidification pattern was observed characterizing the down flow zones. In a second series of experiments the initial state was a solidified material between two rigid plates. The lower plate was heated at a constant rate for a certain time, so that the melt occupied part of the cell. The upper boundary was thus a solidified material. The convection patterns were impressed on the upper solidified boundary and using an optical treatment Pantaloni and Bailleux deduced that the pattern consisted of hexagons shifted with respect to each other by about 15°. The Rayleigh number based on the smallest cell size had a value of about 2300.

Onset of convection in a molten material was also the subject of the paper by W. Hame and U. Müller* (Nuclear Research Centre, Karlsruhe), except that here internal heat sources were considered. A linear stability analysis was carried out for two cases: one, where the molten material was between two rigid plane walls kept at the same fixed temperature T_W below the melting point temperature T_M so that T_M was reached at an interface within the domain. The other was an asymmetric situation with the upper wall kept at constant $T_W < T_M$ and with an adiabatic boundary below. Hame and Müller arrived in both cases at a closed-form eigenvalue problem for the molten part containing the internal Rayleigh number and the Biot number. It was found that lowering T_W decreased both, the critical internal Rayleigh number and the critical wavenumber. The amplification rate of the perturbations increased with latent heat release. The physical properties were treated as constants and hence the limiting case of $T_W = T_M$ reduced respectively to the problems studied experimentally by Kulacki & Goldstein (1972) and by Tritton & Zarraga (1967).

5. Rotating systems

The important role of convection flows in planetary and stellar systems has been, for a long time, a primary motivation for theoretical and experimental studies of convection. Some of the problems of convection in planetary and stellar interiors are strongly influenced by the effects of rotation, and, for this reason, a special session of the Colloquium was devoted to rotating systems. F. H. Busse opened this session with a review of convection in rotating and non-rotating spherical shells subject to spherically symmetric gravity and heat source distributions. The horizontal isotropy of the non-rotating shell contrasts sharply with the distinguished direction of the axis in the rotating shell. Accordingly, the main theoretical task is the removal of the degeneracy of patterns of convection in the former case, while a number of interesting, nonlinear effects associated with the anisotropy can be explored in the latter case. Most of the work discussed is available in the literature (Busse 1975*a*, *b*, 1977; Busse & Carrigan 1976; Gillman 1977; Soward, 1977). New theoretical and experimental results on the generation of a differential rotation by convection in a rotating annulus were also reported (Busse & Hood 1979). The remarks by Busse on convection in rotating spheres were amplified by C. R. Carrigan* (University of Cambridge), who described the results of his Ph.D. thesis research (1977) on convection in a rotating sphere. The laboratory experiments demonstrate that the centrifugal force can be used quite successfully to simulate convection in rotating, self-gravitating spheres (figure 2, plate 2). The experimental observations offer the opportunity to investigate the nonlinear properties of convection which are not easily accessible to theoretical analysis. The observed spiral structure of convection columns is such a property. It appears to be in qualitative agreement with a particular solution of Soward's (1977) nonlinear analysis.

Baroclinic† waves differ from thermal convection in that they occur in the case of a stably stratified density distribution, but otherwise they have much in common with

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† Baroclinic means that the density gradient $\nabla\rho$ is not parallel to direction of gravity. This gives rise to vorticity through $\nabla p \times \nabla\rho$ (p being the pressure).

thermal convection in rotating systems. Nonlinear properties of baroclinic waves, in particular the side-band phenomena, have recently been studied by Hide, Mason & Plumb (1977) in an annulus experiment. The paper by I. N. James and P. R. Jonas* (Meteorological Office, Bracknell) presented a new theoretical model for the nonlinear energy exchange leading to the appearance of the side bands. Experimental observations of the radial structure of the different modes are not consistent with the mechanism of Hide *et al.* and favour the more complicated energy exchange of the new model.

The properties of baroclinic waves in the presence of a nonuniform magnetic field were discussed by Y. Fautrelle* (University of Grenoble). It is known that a uniform weak magnetic field has a damping effect on baroclinic waves and Fautrelle pointed out that a non-uniform weak magnetic field has a destabilizing influence on high wavenumber asymmetric disturbances (Fautrelle 1978). For stronger magnetic fields a higher degree of asymmetry would be necessary. This problem is of importance because dynamos based on baroclinic waves have been advocated for the generation of the earth's as well as the sun's magnetic fields. Because of the nearly two-dimensional nature of baroclinic waves, it is difficult to construct a dynamo mechanism (Braginskii & Roberts 1975). The influence of a magnetic field on baroclinic waves may be of considerable importance in the postulated stably stratified region of the outer core of the earth.

Baroclinicity is also a major source of motion in the stably stratified interiors of stars. Although the motions are extremely slow, they are turbulent because of the huge dimensions involved and they can significantly affect the chemical composition and evolution of stars. The difficulties encountered by astrophysicists in trying to explain the abundances of Li, ^{13}C and other elements involved in the thermonuclear reaction in stars were vividly described by E. Schatzman* (Nice Observatory) who also outlined an approach for the determination of the appropriate turbulent diffusivities of elements in stellar interiors (Schatzman 1977). He finds that the slow turbulence in the stably stratified region of a rotating star leads to diffusivities that exceed the molecular values by a factor of a few times 10^2. Schatzman envisions that the shear caused by the angular momentum loss owing to the magnetic stresses acting on the stellar wind is the origin of the turbulent motions.

Since Taylor vortex flow is closely related to Rayleigh–Bénard convection, it was appropriate to include the presentation of some recent work on that topic in the Colloquium. M. Wimmer described the theoretical simulations by K. G. Roessner* (University of Karlsruhe) of the flow between two concentric spheres, the inner of which is rotating. Depending on the form of acceleration by which a certain steady rotation rate is approached, different states of motion are achieved exhibiting one, two or more steady or oscillating vortices between equator and poles. Even the case of an axisymmetric double cell can be realized. The computational results, which were aptly demonstrated in a movie, are in good agreement with Wimmer's (1976) own experimental realizations of the same problem.

The problem of the flow in a spherical gap has the advantage that the boundary conditions are well defined, which can cause difficulties in the interpretation of the onset of Taylor vortex flow between concentric cylinders. The end-effects play a significant role in most experiments and their influence becomes even more pronounced in the case of combined axial and azimuthal flow. This latter problem was addressed by

B. W. Martin* (University of Wales, Cardiff), who described experimental studies of the instability owing to the developing azimuthal motion of an axial flow entering a cylindrical annulus, the inner wall of which is rotating. Details on the experimental apparatus and the measurements can be found in the recent paper by Gravas & Martin (1978).

The influence of a vertical magnetic field on the onset of convection is in many respects similar to that of rotation about a vertical axis. But when the convective motion reaches finite amplitudes such that the magnetic Reynolds number R_m is no longer small, the magnetohydrodynamics of convection exhibits a rich variety of rather unique phenomena. Since there has been little experimental research on convection in the presence of magnetic fields, most theoretical studies of the nonlinear magnetohydrodynamics of convection have been motivated by processes observed on the surface of the sun. Recent work by Galloway, Proctor & Weiss (1978) on the compression of flux tubes in an axisymmetric convection cell driven by a horizontal temperature gradient was described by the first of these authors. The geometrical configuration is of crucial importance: in the weak field regime the magnetic field amplification grows proportional to R_m at the centre of an axisymmetric convection cell, while the amplification rate is proportional to $R_m^{\frac{1}{2}}$ for two-dimensional rolls. At larger strengths of the imposed magnetic field, the amplification is limited by the Lorentz force. Simple expressions for the maximum field strength can be derived which agree well with the results of numerical computations.

The axisymmetric problem offers the advantage that a simple boundary layer analysis is applicable which does not seem to work in the two-dimensional case. M. R. E. Proctor* (University of Cambridge) reported on his joint work with D. J. Galloway* (University of Sussex; see Proctor & Galloway 1979) on the dependence on the Rayleigh number of the amplitude of convection in a cylindrical box heated from below in the presence of an external magnetic field. Using the boundary-layer method it is found that subcritical, finite amplitude instability is possible for a sufficiently strong magnetic field. The results were compared and contrasted with the two-dimensional calculations of Busse (1975*a*, *b*).

6. Other driving forces

Driving mechanisms other than of gravitational origin are of interest in many situations. The case of electrically induced body force which has recently drawn attention gives rise to electrohydrodynamic convection problems bearing analogies (but also marked differences) with the Rayleigh–Bénard problem. Another field of investigation is provided by instabilities in liquid crystals, an example of which was already considered in chapter 4. The interaction of the anisotropic properties of these media with an electric field or a shear gives rise to new instability mechanisms. It was thus of interest to have a session devoted to these relatively new topics.

In isothermal fluids of low electric conductivity the electrical body force reduces to the Coulomb force resulting from the action of the electric field E on the induced space charges. For destabilizing charge density distributions, instability sets in above a critical value of a space charge Rayleigh number $T = M^2 P_e$ with $M^2 = \epsilon/\rho K^2$, $P_e = KV/\nu$, where ϵ is the permittivity, K the ionic mobility, ν the kinematic viscosity and ρ the fluid density. E. J. Hopfinger* (University of Grenoble) introduced electroconvection by discussing the essential developments of the work done by the Grenoble group during the last decade on the problem of unipolar injection in an insulating liquid. In particular he presented theoretical and experimental results on the existence of both linear and nonlinear stability criteria (Atten & Lacroix 1979) and on the convection régimes as the voltage is further increased (Lacroix, Atten & Hopfinger 1975). The electrical nature of the unipolar injection problem makes its study in transient conditions easier than the thermal problem. A recent study by E. J. Hopfinger, P. Atten and J. C. Lacroix* (Laboratoire d'Électrostatique, Grenoble) was concerned with the temporal development of convection when a step voltage is applied. It has been shown that the sequence of régimes observed in time was closely related to the different flow régimes under steady conditions. After instability is manifested the velocity of the convection front and the current increase exponentially as long as the Reynolds number (*Re*) is lower than 10 (viscous régime). In the turbulent state ($Re > 10$) the front moves with constant velocity proportional to MKE. These observations were illustrated by a film showing the different features for various values of the parameter M taking here the role of the Prandtl number.

The paper presented by J. L. Sproston* (University of Liverpool) was also concerned with electroconvection but oriented to engineering applications. The instability and convection phenomena were clearly driven by the Coulomb force but the use of a system for which the electrical conduction and the electrode injection conditions were not well defined and controlled prevented the author from quantitative comparison with existing theories.

Electrohydrodynamic phenomena in nematic liquid crystals involve a different mechanism. Usually injection from the electrodes play but a negligible role, the medium being sufficiently conducting. R. Ribotta* (University of South Paris, Orsay) recalled that the driving force arises from the anisotropic properties of the liquid crystals; in particular for material with negative dielectric anisotropy ($\epsilon_{\parallel} < \epsilon_{\perp}$) a distortion of the director induces a space charge related to the anisotropy of electric conductivity and for a planar orientation (director parallel to the electrodes) the Coulomb force tends to amplify this orientational disturbance. Instability sets in above some voltage threshold V_c for d.c. or low frequency a.c. fields (Dubois-Violette *et al.* 1971). Using a cell with two adjacent regions, one supercritical and the other subcritical, Ribotta studied the influence length, i.e. the characteristic length for the exponential decrease of the amplitude of the rolls in the subcritical region (figure 3, plate 1). Here again the classical law was obtained for the influence length which varied like $(V_c - V)^{-\frac{1}{2}}$. A movie showed the rolls in the subcritical region and numerous convection régimes and pattern behaviours as voltage is increased.

Another class of instabilities exhibited by these nematics are 'shear flow' instabilities when the director is normal to both the velocity and its gradient. P. Manneville* (C.E.N. Saclay, Gif-sur-Yvette) recalled that in this case the driving mechanism has nothing in common with the Reynolds stresses but relates again to the special properties of these anisotropic liquids here reflected in the viscous stress tensor (see Jenkins 1978). The coupling between orientation distortions and viscous torques turns out to be destabilizing and a secondary flow sets in above a critical shear rate. In the case of plane Couette flow two different modes of instability, homogeneous and spatially periodic, were obtained (Pieranski & Guyon 1974). Manneville then reported on the stability study of the cylindrical Couette flow of nematics with the unperturbed director parallel to the rotation axis and with positive and negative differential rotations. The Coriolis force had no primary role but only interplayed with the different couplings specific to nematics. He pointed out that homogeneous or roll secondary flows emerge, depending on the sign of a particular viscosity coefficient. The critical shear generally increases with the mean rotation rate (Dubois-Violette & Manneville 1978).

7. General theories and transition to turbulence

Convection in a layer heated from below has received special attention in recent years because it exhibits the onset of turbulence in a relatively simple manner. New experimental techniques such as laser-Doppler velocimetry and the highly accurate calorimetric methods developed for the research on liquid helium have been applied to the problem of transition to turbulence in convection. The new results obtained by this research were among the most interesting ones presented at the Colloquium.

The theoretical problem of the transition to turbulence was reviewed by D. D. Joseph* (University of Minnesota). The theory is best developed in fluid systems such as the Taylor vortex and Bénard convection problems where a sequence of supercritical bifurcations leads to the onset of turbulence. The older picture of Landau and Hopf envisioned a gradual onset of turbulence by bifurcations of states of motion with increasing complexity. In the simplest form each bifurcation would add a new fundamental frequency to a spectrum of discrete frequencies until the spectrum could no longer be distinguished from a continuous one. Recent experimental evidence seems to favour the idea of Ruelle & Takens (1971), who maintain that the bifurcation with the third fundamental frequency is not realized in general and that systems approach, instead, a 'strange attractor' in the phase space, which is characterized by an aperiodic time dependence with a continuous spectrum.

Many hydrodynamic systems such as Poiseuille and plane Couette flow exhibit subcritical bifurcations. In these cases sudden transitions from laminar to turbulent flows occur and 'brute force' numerical computations appear to be the only way to obtain a theoretical description of the realized turbulent state. Numerical simulations using a mean-field approximation were done some years ago (Zahn *et al.* 1974), but only recently have Orszag & Krell (1978) been able to integrate the three-dimensional time-dependent problem without using simplifying assumptions. An extended version of Joseph's review lecture will be published in the near future (Joseph, 1979).

The problem of bifurcation and the transition between states with different kinds of order was discussed from a very general point of view by H. Haken* (University of Stuttgart). He motivated his approach by the observation that similar forms of bifurcation occur in lasers, fluids, chemical reactions, morphogenesis in biological systems, and even in the dynamics of social groups (Haken 1975). The mathematical method used by Haken represents an application of the classical tools of bifurcation theory such as Floquet's theory to manifolds of solutions. This allows the consideration of stochastic forces in the problems and opens the possibility for application of the theory to transitions between turbulent states of a system.

A general approach to problems of hydrodynamic instabilities and bifurcation from a different point of view was described in the paper by I. Prigogine* (Free University of Brussels). It presented a generalization of the thermodynamics of irreversible processes and Einstein's theory of fluctuations. Unfortunately, questions raised by the paper, which was read by J. C. Legros, could not be discussed at the Colloquium because the author was unable to attend the meeting.

The problem of the bifurcation from a periodic solution into an invariant two-dimensional torus in the phase space was addressed by G. Iooss* (University of Nice), who reported an analysis done in collaboration with D. D. Joseph. The study was motivated by the experimental observation by Gollub & Swinney (1975) of the transition from time-periodic motion to quasi-periodic motion in the Taylor vortex flow between coaxial cylinders. In his talk, Iooss discussed the bifurcations of quasi-periodic solutions of a system of differential equations and their topological properties in relation to some classical theorems. The mathematical framework provided by different topological possibilities can be used for the interpretation of the experimental observations.

A detailed experimental investigation by A. Bouabdallah and G. Cognet* (Laboratoire d'Énergétique et de Mécanique, Nancy) of the transition to turbulence in the case of Taylor vortex flow was presented by the latter author. The velocity was measured by an electrochemical method. The results are essentially in agreement with those obtained by Gollub & Swinney (1975) using laser-Doppler velocimetry. The new experiments were carried out for three different radius ratios of the cylinders. By changing the axial position of the electrodes the time dependence in different parts of the wavy or turbulent Taylor vortex could be observed. The data showed that the peaks in the spectrum of the time dependence were less pronounced at places of low velocity than at places of high velocity. Further analysis of the data may help to understand the different mechanisms associated with the two fundamental frequencies.

The evolution of the time dependence of convection in a layer heated from below was the subject of the papers by Bergé, by Gollub & Benson, and by Maurer & Libchaber. Using laser-Doppler velocimetry, P. Bergé* (C.E.N. Saclay, Gif-sur-Yvette) has been able to make detailed measurements of the amplitude of the various Fourier components of the convection velocity field as a function of the Rayleigh number. The comparison with theoretical computations shows good agreement. Of particular interest is the observation of the onset of time dependence in convection layers with two different aspect ratios. In the case of the large aspect ratio with Prandtl number $P = 130$, a slowly varying, but erratic, time dependence was observed when the Rayleigh number R exceeds 20 times the critical value R_c. Not until R reached $30R_c$ could some intermittent, approximately periodic oscillations be observed. An inverse sequence of events is typical for layers with a small aspect ratio. In the particular case studied by Bergé, a fluid with $P = 500$ was used and oscillations were first observed for $R = 242R_c$. Apart from the oscillations, the pattern of convection remained steady in this case. When the Rayleigh number was increased to $286R_c$, changes in the pattern occurred, the frequency peaks broadened, and a continuous spectrum of low-frequency noise developed. The critical aspect ratio separating the two different forms of the time-dependence evolution seems to be close to 10 in high Prandtl number fluids.

J. P. Gollub* (Haverford College, Pennsylvania) emphasized in his talk the similarity between the evolution of the time dependence in Taylor vortex flow (Gollub & Swinney 1975) and in low-aspect-ratio convection (Fenstermacher *et al.* 1978). In both cases, a sequence of events starting with steady roll-like motions and proceeding to states of periodic, quasiperiodic and finally aperiodic flows is observed as the Rayleigh or Taylor number is increased. There have been never more than two fundamental frequencies observed, and the transition from quasi-periodic to aperiodic flow is well defined by the appearance of broad band noise, although the actual value of the Rayleigh number at which the transition takes place may depend on the history of the experiment. The transition occurs at rather low values of the Rayleigh number if the temperature at boundaries is slightly modulated with the difference frequencies of the two fundamental frequencies of the layer.

The experiment described in the third paper on time-dependent convection is based on the same method as that of Ahlers (1974) and Ahlers & Behringer (1978) on convection in liquid helium. In contrast to Ahlers, J. Maurer & A. Libchaber* (E.N.S., Paris) used a local probe to measure the time dependence of convection in liquid helium. As in the experiments reported by Bergé, the evolution of the time dependence depends on the aspect ratio of the convection layer. A layer with the aspect ratio $\Gamma = 2$ exhibits well-defined periodic oscillations for $R \geqslant 2{\cdot}5R_c$, while low-frequency noise appears at $R \approx 2R_c$ when $\Gamma = 6$ is used. The oscillations occurring in the latter case at $R \approx 3{\cdot}8R_c$ are not well defined and correspond to broad peaks in the frequency spectrum (Libchaber & Maurer 1978). Since helium has a Prandtl number of about 0·6, it must be concluded that the critical aspect ratio Γ_c separating the two different ways in which the time dependence of convection evolves depends on the Prandtl number. Just recently Libchaber has started to use two probes to obtain spatial correlation measurements.

While the onset of oscillations in convection is reasonably well understood, at least in fluids of low or moderate Prandtl numbers (Clever & Busse 1974), the slowly varying time-dependence associated with convection in layers of large aspect ratio has not yet found a precise theoretical description. Y. Pomeau* (C.E.N. Saclay, Gif-sur-Yvette) outlined a mathematical approach to the problem and suggested a connexion between the slow variations in time and the increase of the wavelength of convection observed at increasing Rayleigh number. More detailed calculations are required, however, to allow a quantitative comparison with the experimental observations.

8. Conclusion

It became clear during the colloquium that important progress has been made in convection during the past few years and the activity in this field has been more intense than we expected when planning this colloquium. This activity has been stimulated by new experimental techniques and by the increasing importance of new phenomena and applications, such as doubly diffusive convection, convection in liquid crystals, electroconvection and convection in solar-energy storage systems. Besides, in this field of research theory and experiments have progressed by mutual stimulation and the sixth session of this colloquium (see §7) has beautifully demonstrated this interplay between theory and experiment on the difficult problem of transition to turbulence in Bénard convection and Taylor vortex flow. Present results seem to favour the idea of Ruelle–Takens and there is hope that in the coming few years the mechanism of transition might be fully understood. Experiments of this kind in other simple systems would be of great value and electroconvection and convection in liquid crystals seem to be promising. Refined experimental techniques have also made it possible to demonstrate the analogy between the onset of instability in the Rayleigh–Bénard problem and second-order phase transition, a property that provides new perspectives for hydrodynamic instabilities.

Detailed conclusions are apparent from the discussions of the individual papers but some general tendencies and needs for future work that have emerged from the meeting are mentioned here.

(i) End effects and external excitations play a significant role in the onset of instability and the subsequent transitions and this causes difficulties in the interpretation of experimental results. Theoretical work on this problem, in particular bifurcation theories, have been and will remain of great help in the understanding of these effects.

(ii) The transition from steady to a time dependent convection régime in the R–P diagram at high Prandtl numbers has received much attention but more work is needed in order to clarify transition criteria. It would also be of interest to study such transitions in relatively low Prandtl number fluids and further experiments in liquid helium ($P \approx 0{\cdot}6$) and perhaps in mercury should be encouraged.

(iii) In geophysical situations well-organized convection patterns can be observed even at very high Rayleigh numbers (in cloud layers, for instance). A possibly related phenomenon is the kinks in the dependence of the heat transport on the Rayleigh number found by Malkus (1954). It is likely that further experimental studies of high Rayleigh number convection will reveal additional regular features of what superficially appears to be a highly chaotic system.

(iv) The influence of shear on thermal convection is of interest in many situations, the most striking one being the appearance of cloud streets in the planetary boundary layer. The problem has been studied over decades (Kelly 1978) but secondary instabilities (waviness) of the longitudinal rolls and their possible correlation with three-dimensional structures in the convection rolls have only recently been considered. More detailed experimental observations would be very useful.

(v) There are presently two kinds of computational models for turbulent convection: one-point closure models and modal equation techniques. The latter are more elegant and the physical processes are more tractable, but there is some inconsistency in treating the vertical scale in detail while using only one horizontal wavenumber. The turbulence model is certainly more adequate when numerical values are desired but the approach is cumbersome and physical processes seem sometimes masked. There is need for a confrontation and further development.

These are only some aspects and we have already mentioned the considerable progress made on the problem of transition to turbulence. Furthermore, the very fascinating problems associated with convection in planetary and stellar systems where rotation and/or magnetic field are of importance continue to be a primary motivation for studies of convection.

REFERENCES

Acheson, D. J. 1976 On over-reflexion. *J. Fluid Mech.* **77**, 433–472.

Acheson, D. J.* Hydrodynamic instabilities provoked by bottom-heavy gradients.

Acheson, D. J. 1978 On the stability of toroidal magnetic fields and differential rotation in stars. *Phil. Trans. Roy. Soc.* A **289**, 459–500.

Adrian, R. J. 1975 Turbulent convection in water over ice. *J. Fluid Mech.* **69**, 753–781.

Ahlers, G. 1974 Low temperature studies of the Rayleigh–Bénard instability and turbulence. *Phys. Rev. Lett.* **33**, 1185–1188.

Ahlers, G. 1975 The Rayleigh–Bénard instability at helium temperatures. In *Fluctuations, Instabilities and Phase Transitions* (ed. T. Riste), vol. B 11, pp. 181–193. New York: Plenum Press.

Ahlers, G. & Behringer, R. P. 1978 Evolution of turbulence from the Rayleigh–Bénard instability. *Phys. Rev. Lett.* **40**, 712–716.

André, J. C., De Moor, G., Lacarrère, P. & Du Vachat, R. 1976 Turbulence approximation for inhomogeneous flows. I. The clipping approximation. *J. Atmos. Sci.* **33**, 476–481.

André, J. C., De Moor, G., Lacarrère, P. & Du Vachat, R.* Convection turbulente. Simulation numérique d'une expérience de laboratoire.

Antoranz, J. C. & Velarde, M. G. 1978 Soret-driven convective instability with rotation. *Phys. Lett.* A **65**, 377–379.

Atten, P. & Lacroix, J. C. 1979 Nonlinear hydrodynamic stability of liquids subjected to unipolar injection. *J. Mécan.* (in press).

Azouni, M. A. 1977 Instabilités thermoconvectives entre 0 °C et 4 °C. *Lett. Heat Mass Transfer* **4**, 445–452.

Azouni, M. A.* Hydrodynamic instabilities in a fully confined water layer under its density maximum.

Baranowski, B.* The electrochemical analogue of Bénard instability.

Benjamin, T. B. 1978 Bifurcation phenomena in steady flows of a viscous fluid. *Proc. Roy. Soc.* A **359**, 1–26.

Berge, P.* Experimental evidence of the behaviour of convective velocity in Rayleigh–Bénard instability: Different transitions toward turbulence.

Berkovsky, B. M., Fertman, V. E., Sinitsyn, A. K. & Barkov, Yu. I. 1978 A theoretical and experimental study of thermal disturbances propagating in a fluid layer heated from below. *J. Fluid Mech.* **89**, 173–190.

Bois, P. A.* Instabilité linéaire convective dans l'atmosphère. Théorie asymptotique.

Bouabdallah, A. & Cognet, G.* Instabilités et turbulence dans l'écoulement de Taylor–Couette.

Braginskii, S. I. & Roberts, P. H. 1975 Magnetic field generation by baroclinic waves. *Proc. Roy. Soc.* A **347**, 125–140.

Busse, F. H. 1967 The stability of finite amplitude cellular convection and its relation to an extremum principle. *J. Fluid Mech.* **30**, 625–649.

Busse, F. H. 1975*a* Patterns of convection in spherical shells. *J. Fluid Mech.* **72**, 67–85.

Busse, F. H. 1975*b* Nonlinear interaction of magnetic field and convection. *J. Fluid Mech.* **71**, 193–206.

Busse, F. H. 1977 Convection in rotating stars. In *Problems of Stellar Convection* (ed. E. A. Spiegel & J. P. Zahn), Lecture Notes in Physics, no. 71, pp. 156–175. Springer.

Busse, F. H.* Convection in planetary cores.

Busse, F. H. & Carrigan, C. R. 1976 Laboratory simulation of thermal convection in rotating planets and stars. *Science* **191**, 81–83.

Busse, F. H. & Hood, L. 1979 Generation of differential rotation by convection. (In preparation.)

Busse, F. H. & Whitehead, J. A. 1974 Oscillatory and collective instabilities in large Prandtl number convection. *J. Fluid Mech.* **66**, 67–80.

Caltagirone, J. P. 1976 Thermoconvective instabilities in a porous medium bounded by two concentric horizontal cylinders. *J. Fluid Mech.* **76**, 337–362.

Caltagirone, J. P. & Mojtabi, A.* Stabilité linéaire et énergétique de l'écoulement de convection naturelle dans une couche poreuse annulaire.

Carrigan, C. R. 1977 A study of buoyancy driven flows in rotating fluids motivated by planetary applications. Ph.D. Thesis, University of California.

Carrigan, C. R. & Busse, F. H.* A laboratory study of convection in a rapidly rotating spherical layer.

Castillo, J. L. & Velarde, M. G. 1978 Thermal diffusion and the Marangoni–Bénard instability of a two-component fluid layer heated from below. *Phys. Lett.* A **66**, 489–491.

Chandrasekhar, S. 1961 *Hydrodynamic and Hydromagnetic Stability.* Oxford University Press.

Childress, S., Levandowsky, M. & Spiegel, E. A. 1975 Pattern formation in a suspension of swimming micro-organisms: Equations and stability theory. *J. Fluid Mech.* **63**, 591–613.

Clever, R. M.* Instabilities of longitudinal convection rolls with shear.

Clever, R. M. & Busse, F. H. 1974 Transition to time dependent convection. *J. Fluid Mech.* **65**, 625–645.

Clever, R. M. & Busse, F. H. 1977 Instabilities of longitudinal convectien rolls in an inclined layer. *J. Fluid Mech.* **81**, 107–127.

Clever, R. M., Busse, F. H. & Kelly, R. E. 1977 Instabilities of longitudinal convection rolls in Couette flow. *J. Appl. Math. Phys.* **28**, 771–783.

Donnelly, R. J., Schwartz, K. W. & Roberts, P. H. 1965 Experiments on the stability of viscous flow between rotating cylinders. IV Finite amplitude experiments. *Proc. Roy. Soc.* A **283**, 531–000.

Dubois, M. & Wesfreid, J.* Experiments on Rayleigh–Bénard convection in simple fluids: Critical properties near threshold and penetration effects.

Dubois-Violette, E., de Gennes, P. G. & Parodi, O. 1971 Hydrodynamic instabilities of nematic liquid crystal under A.C. electric fields. *J. Phys.* **32**, 305–317.

Dubois-Violette, E. & Manneville, P. 1978 Stability of the Couette flow in nematic liquid crystals. *J. Fluid Mech.* **89**, 273–303.

Faust, K. M. & Plate, E. J.* Turbulent penetrative convection.

Fautrelle, Y. 1978 Baroclinic waves in the presence of a magnetic field. *J. Méc.* **17**, 1–34.

Fautrelle, Y.* Baroclinic instability in the presence of a magnetic field.

Fenstermacher, P. R., Swinney, H. L., Benson, S. V. & Gollub, J. P. 1978 In *Bifurcation Theory and Applications in Scientific Disciplines* (ed. O. Gurel and O. E. Rössler). New York Academy of Sciences.

Galloway, D. J., Proctor, M. R. E. & Weiss, N. O. 1978 Magnetic flux ropes and convection. *J. Fluid Mech.* **87**, 243–261.

Galloway, D. J.* The effects of geometry on convection with magnetic fluxropes.

Giglio, M. & Vendramini, A. 1977 Buoyancy-driven instability in a dilute solution of macromolecules. *Phys. Rev. Lett.* **39**, 1014–1017.

Giglio, M. & Vendramini, A.* Soret-driven instability in a dilute solution of macromolecules.

Gill, A. E. & Davey, A. 1969 Instabilities of a buoyancy driven system. *J. Fluid Mech.* **35**, 775–798.

Gilman, P. A. 1977 Nonlinear dynamics of Boussinesq convection in a deep rotating spherical shell, I. *Geophys. Astrophys. Fluid Dyn.* **8**, 93–136.

Gollub, J. P. & Benson, S. V.* Laser-Doppler studies of convective instabilities and the transition to turbulence.

Gollub, J. P. & Swinney, H. L. 1975 Onset of turbulence in a rotating fluid. *Phys. Rev. Lett.* **35**, 927–930.

Gough, D. O. 1969 The anelastic approximation for thermal convection. *J. Atmos. Sci.* **26**, 448–456.

Gough, D. O., Spiegel, E. A. & Toomre, J. 1975 Modal equations for cellular convection. *J. Fluid Mech.* **68**, 695–719.

Graham, R. 1975 Macroscopic theory of fluctuations and instabilities in optics and hydrodynamics. In *Fluctuations, Instabilities and Phase Transitions* (ed. T. Riste), vol. B 11, 215–293. New York: Plenum Press.

Gravas, N. & Martin, B. W. 1978 Instability of viscous axial flow in annuli having a rotating linear cylinder. *J. Fluid Mech.* **86**, 385–394.

Guyon, E., Pieranski, P. & Salan, J. 1978 Cristaux liquides – Convection oscillante dans les nématiques. *C. r. hebd. séanc. Acad. Sci., Paris*, B **287**, 41–43.

Guyon, E., Pieranski, P. & Salan, J.* Heat convection in nematics in the presence of fields.

HAKEN, H. 1975 Cooperative phenomena in systems far from thermal equilibrium and in nonphysical systems. *Rev. Mod. Phys.* **47**, 67–121.

HAKEN, H.* Bifurcation of limit cycles and quasiperiodic flows in continuous media.

HALL, P. & WALTON, I. C. 1977 The smooth transition to a convective régime in a two-dimensional box. *Proc. Roy. Soc.* A **358**, 199–221.

HAME, W. & MULLER, U.* Free convective flow in a molten pool with internal heat sources and solidified boundaries.

HART, J. E. 1971 Transition to a wavy vortex régime in convective flow between inclined plates. *J. Fluid Mech.* **48**, 265–271.

HIDE, R., MASON, P. J. & PLUMB, R. A. 1977 Thermal convection in a rotating fluid subject to a horizontal temperature gradient: Spatial and temporal characteristics of fully developed baroclinic waves. *J. atmos. Sci.* **34**, 930–950.

HOPFINGER, E. J., ATTEN, P. & LACROIX, J. C.* Coulomb force induced instability and convection in dielectric liquids.

HOWARD, L. N. 1966 Convection at high Rayleigh number. In *Proc. 11th Int. Congress of Applied Mechanics*, pp. 1109–1115. Springer.

HUPPERT, H. E. & LINDEN, P. F.* Heating a stable salinity gradient from below.

IOOSS, G.* Properties of a flow bifurcating from a self-oscillating one.

JAMES, I. N. & JONAS, P. R.* Recent combined laboratory and numerical studies of baroclinic waves in an annulus.

JENKINS, J. T. 1978 Flows of nematic liquid crystals. *Ann. Rev. Fluid Mech.* **10**, 197–219.

JONES, C. A. & MOORE, D. R.* The stability of axisymmetric convection.

JONES, C. A., MOORE, D. R. & WEISS, N. O. 1976 Axisymmetirc convection in a cylinder. *J. Fluid Mech.* **73**, 353–388.

JOSEPH, D. D. 1976 Stability of fluid motions. I and II. *Springer Tracts in Natural Philosophy*, vols. 27 and 28. Springer.

JOSEPH, D. D. 1979 Hydrodynamic stability and birfurcation. To appear in *Hydrodynamic Instabilities and the Transition to Turbulence*, ed. Swinney and Gollub, Topics in Current Physics. Springer.

JOSEPH, D. D.* Review of repeated and direct bifurcations into turbulence.

KELLY, R. E. 1978 The onset and development of Rayleigh–Bénard convection in shear flows: A review. In *Proc. Int. Conf. Phys. Chem. and Hydrodynamics*, pp. 65–79. Hemisphere Publ. Corp.

KLAPISZ, C., STRAUSS, B., VAN GRUNDERBEECK, P. & WEILL, A.* Convection and similarity in the free convection atmospheric layer.

KRISHNAMURTI, R. 1968*a* Finite amplitude convection with changing mean temperature. 1. Theory. *J. Fluid Mech.* **33**, 445–455.

KRISHNAMURTI, R. 1968*b* Finite amplitude convection with changing mean temperature. 2. An experimental test of the theory. *J. Fluid Mech.* **33**, 457–463.

KRISHNAMURTI, R. 1970 On the transition to turbulent convection, II. The transition to time-dependent flow. *J. Fluid Mech.* **42**, 309–320.

KRISHNAMURTI, R. 1973 Some further studies on the transition to turbulent convection. *J. Fluid Mech.* **60**, 285–303.

KRISHNAMURTI, R. 1975*a* On cellular cloud patterns. Part 2: Laboratory model. *J. Atmos. Sci.* **32**, 1364–1372.

KRISHNAMURTI, R. 1975*b* On cellular cloud patterns. Part 3: Applicability of the mathematical and laboratory models. *J. Atmos. Sci.* **32**, 1373–1383.

KRISHNAMURTI, R.* A review of convective instability with examples from atmospheric and biological convection.

KULACKI, F. A. & GOLDSTEIN, R. J. 1972 Thermal convection in horizontal fluid layer with uniform volumetric energy sources. *J. Fluid Mech.* **55**, 271–287.

LACROIX, J. C., ATTEN, P. & HOPFINGER, E. J. 1975 Electroconvection in a dielectric liquid layer subjected to unipolar injection. *J. Fluid Mech.* **69**, 539–563.

Latour, J., Spiegel, E. A., Toomre, J. & Zahn, J. P. 1976 Stellar convection theory I. The anelastic modal equations. *Astrophys. J.* **207**, 233–243.

Latour, J., Toomre, J. & Zahn, J. P.* Nonlinear penetrative Boussinesq convection.

Legros, J. Cl., Van Hook, W. A. & Thomaes, G. 1968 Convection and thermal diffusion in a solution heated from below. *Chem. Phys. Lett.* **2**, 249–250.

Legros, J. Cl. & Platten, J. K.* Le problème de Bénard à deux constituants: théorie non-linéaire, parois rigides.

Legros, J. Cl. & Platten, J. K.* 1978 Les instabilités hydrodynamiques en convection libre, forcée et mixte. *Lecture Notes in Physics*, vol. 72. Springer.

Lekkerkerker, H. N. W. 1977 Oscillatory convective instabilities in nematic liquid crystals. *J. Phys. Lett.* L **38**, 277–281.

Libchaber, A. & Maurer, J. 1978 Local probe in a Rayleigh–Bénard experiment in liquid helium. *J. Phys. Lett.* L **39**, 369–372.

Linde, H. & Schwartz, P.* Dissipative structures and nonlinear kinetics of the Marangoni instability.

Malkus, W. V. R. 1954 Discrete transitions in turbulent convection. *Proc. Roy. Soc.* A **225**, 185–195.

Manneville, P.* Stability on the cylindrical Couette flow in nematic liquid crystals.

Martin, B. W.* Instability of viscous axial flow in an annulus with a rotating inner cylinder.

Massaguer, J. M. & Zahn, J. P.* Nonlinear anelastic convection in a highly stratified medium.

Maurer, J. & Libchaber, A.* Local probe in a Rayleigh–Bénard experiment in liquid helium.

McClimans, T. A. & Steen, J. E.* Double diffusion at the freezing point in fjords.

Moore, D. R. & Weiss, N. O. 1973 Nonlinear penetrative convection. *J. Fluid Mech.* **61**, 553–581.

Mucha, Z., Peradzynski, Z. & Baranowski, A. 1977 Instability of continuous optical discharge. *Bull. Acad. Pol. Sci. Ser. Sci. Tech. (Poland)* **25**, 361–367.

Oertel, H.* Time-dependent wave mode convection.

Oertel, H. & Buhler, K. 1978 A special differential interferometer used for heat convection investigations. *Int. J. Heat Mass Transfer* **21**, 1111–1115.

Orszag, S. & Krell, L. 1978 Transition to turbulence in plane Poisseuille and plane Couette flow. Preprint.

Palm, E. 1975 Nonlinear thermal convection. *Ann. Rev. Fluid Mech.* **7**, 39–61.

Palm, E.* On heat and mass flux through snow,

Pantaloni, J. & Bailleux, R.* Experimental results on Bénard convection in molten salt layers: the role of solidification at an open interface and the difference with Bénard–Marangoni instability of an oil layer.

Peradzynski, Z.* Instability of convective flow due to a strong thermal source.

Pieranski, P., Dubois-Violette, E. & Guyon, E. 1973 Heat convection in liquid crystals heated from above. *Phys. Rev. Lett.* **30**, 736–739.

Pieranski, P. & Guyon, E. 1974 Instability of certain shear flows in nematic liquids. *Phys. Rev.* A **9**, 404–417.

Platten, J. K. & Chavepeyer, G. 1977 Non-linear two-dimensional Bénard convection with Soret effect: Free boundaries. *Int. J. Heat Mass Transfer* **20**, 113–122.

Platten, J. K. & Legros, J. Cl.* Simulations numériques de la convection de Rayleigh–Bénard: Exposants critiques, longueur de pénétration, temps d'amortissement.

Pomeau, Y.* Unsteadiness and turbulence near the onset of convection.

Prigogine, I., Legros, J. Cl. & Platten, J. K.* Critère général de stabilité thermodynamique hors de l'équilibre.

Prigogine, I. & Rice, S. A. 1975 Proc. Conf. Instabilities and Dissipative Structures in Hydrodynamics. *Adv. Chem. Physics*, **32**. Wiley.

Proctor, M. R. E.* The bifurcation problem for Rayleigh–Bénard convection with magnetic fields.

Proctor, M. R. E. & Galloway, D. J. 1979 The dynamic effect of the flux ropes on Rayleigh–Bénard convection. *J. Fluid Mech.* In press.

RIBOTTA, R.* Penetration length of a vortex into a subcritical region.

ROESNER, K. G.* Numerical simulation of accelerated spherical gap flow at critical Reynolds numbers.

RUELLE, D. & TAKENS, F. 1971 On the nature of turbulence. *Comm. Math. Phys.* **20**, 167–192.

SCHATZMAN, E. 1977 Turbulent transport and lithium destruction in main sequence stars. *Astron. Astrophys.* **56**, 211.

SCHATZMAN, E.* On the possible existence of turbulence in rotating stars.

SCHINKEL, W. M. M.* Natural convection in inclined air-filled enclosures: An interferometric study.

SCHNEIDER, W. & KECK, H.* Thermoconvective waves.

SCHWABE, D., SCHARMANN, A., PREISSER, F. & OEDER, R. 1978 Experiments on surface tension driven flow in floating zone melting. *J. Cryst. Growth.* **43**, 305–312.

SCHWABE, D., SCHARMANN, A. & PREISSER, F.* Instability of surface tension driven flow.

SOWARD, A. M. 1977 On the finite amplitude thermal instability in a rapidly rotating fluid sphere. *Geophys. Astrophys. Fluid Dyn.* **9**, 19–74.

SPROSTON, J. L.* Electroconvection in insulating liquids.

STEPPELER, J. 1978 Fluid computations using infinitesimal functionals. *Comp. Fluids.* (In the Press.)

STEPPELER, J.* A variational approach to Bénard convection with applications to cumulus convection.

TRITTON, D. J.* High Rayleigh number convection of a very viscous fluid heated from below.

TRITTON, D. J. & ZARRAGA, M. N. 1968 Convection in horizontal layers with internal heat generation: Experiments. *J. Fluid Mech.* **30**, 21–31.

TRUE, H.* Bifurcation and stability of the convective boundary layer at a vertical wall with a stable temperature gradient.

TURNER, J. S. 1965 The coupled turbulent transport of salt and heat across a sharp density interface. *Int. J. Heat Mass Transfer* **8**, 759–767.

TURNER, J. S. 1974 Double-diffusive phenomena. *Ann. Rev. Fluid Mech.* **6**, 37–56.

VELARDE, M. G.* Thermal diffusion and convective instability of binary fluid layers.

VELARDE, M. G. & SCHECHTER, R. S. 1972 Thermal diffusion and convective stability. II. An analysis of the convective fluxes. *Phys. Fluids.* **15**, 1707–1714.

WALTON, I. C.* The smooth transition to a convective regime in a two-dimensional box.

WESFREID, J., BERGE, P. & DUBOIS, M. 1979 Induced pretransitional Rayleigh–Bénard convection. *Phys. Rev.* (to be published).

WESFREID, J., POMEAU, Y., DUBOIS, M., NORMAND, C. & BERGE, P. 1978 Critical effects in Rayleigh–Bénard convection. *J. Phys. Lett.* **39**, 725–731.

WHITEHEAD, J. A. & PARSONS, B. 1978 Observations of convection at Rayleigh numbers up to 760,000 in a fluid with large Prandtl number. *Geophys. Astrophys. Fluid Dyn.* **9**, 201–217.

WHITEHEAD, J. A.* Time dependent high Prandtl number convection.

WILLIS, G. E. & DEARDORFF, J. W. 1970 The oscillatory motions of Rayleigh convection. *J. Fluid Mech.* **44**, 661–672.

WILLIS, G. E. & DEARDORFF, J. W. 1974 A laboratory model of the unstable planetary boundary layer. *J. atmos. Sci.* **31**, 1297–1307.

WIMMER, M. 1976 Experiments on a viscous fluid flow between concentric rotating spheres. *J. Fluid Mech.* **78**, 317–335.

ZAHN, J. P., TOOMRE, J., SPIEGEL, E. A. & GOUGH, D. D. 1974 Nonlinear cellular motions in Poiseuille channel flow. *J. Fluid Mech.* **64**, 319–345.

ZEMAN, O. & LUMLEY, J. L. 1976 Modeling buoyancy driven mixed layers. *J. atmos. Sci.* **33**, 1974–1988.

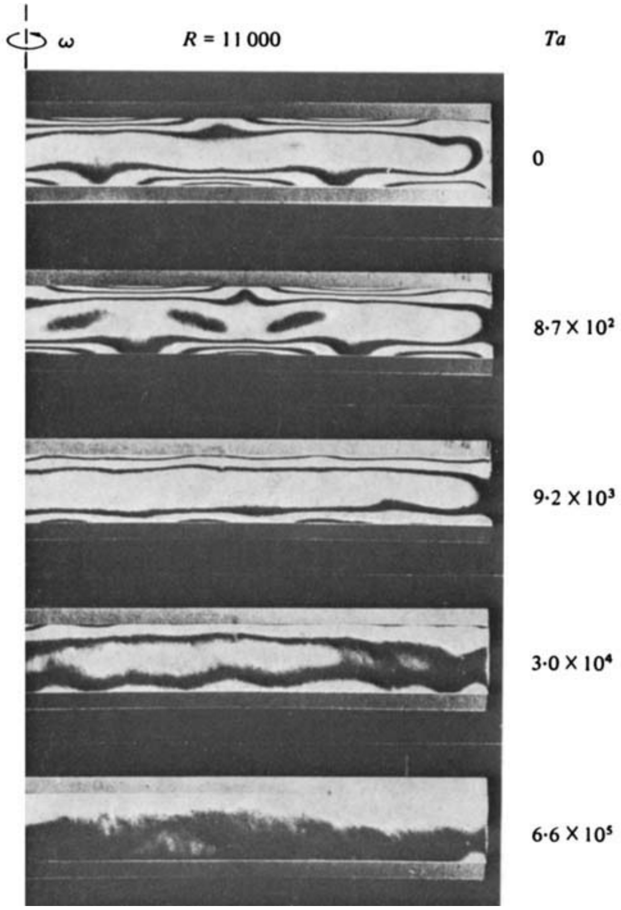
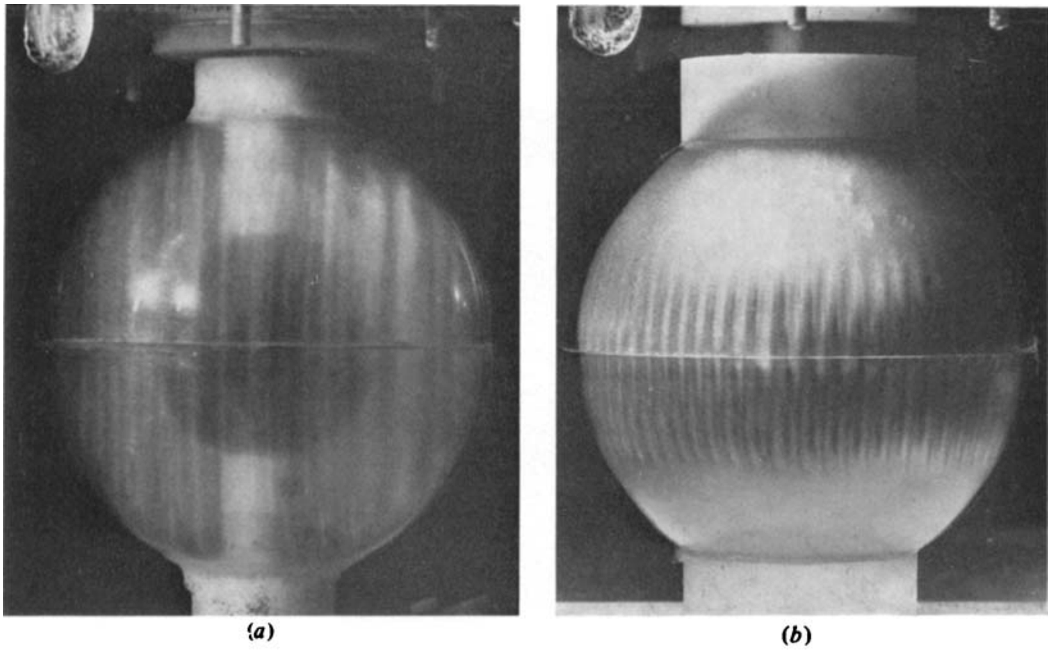


FIGURE 1. Interferogram of convection in nitrogen ($P = 0{\cdot}7$) showing the stabilizing effect of rotation. The Rayleigh number is 11×10^3 and the Taylor number *Ta* was quasi-statically increased up to $6{\cdot}6 \times 10^5$. At $Ta = 6{\cdot}6 \times 10^5$ convection has ceased (by courtesy of Dr Oertel).



(*a*) (*b*)

FIGURE 2. Convection columns aligned with the rotation axis in a rotating spherical shell heated from the outside and cooled from within. (*a*) The ratio between the outer and inner radius is about $\frac{1}{2}$. (*b*) The ratio is approximately $\frac{1}{16}$. Note that here the centrifugal force plays the role of g

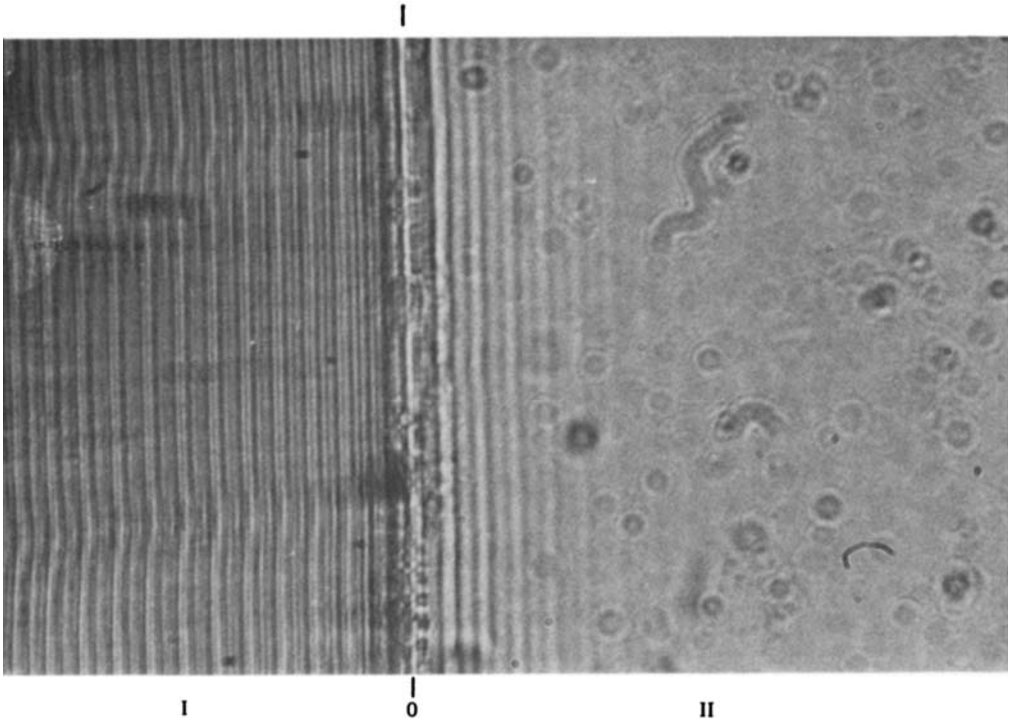


FIGURE 3. A microscope view of convection rolls in a nematic liquid crystal layer, slightly supercritical in region I (left) and subcritical in region II ($V = 0{\cdot}9V_c$). The exponential decrease of the velocity amplitude in region II is apparent from the decrease in contrast with distance from the separation plane 0 (by courtesy of Dr Ribotta).